Star Death
The Case of the Crab Nebula

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Image: 2021-01-12 Nathan Mossaad
Image of Crab Nebula

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1 Introduction

1.1 Overview

What happens when a star dies? What can we learn from its remaining Nebula? In this paper I will take you on a journey of what we can learn with relatively easily obtainable information, without the need to have a lot of prior knowledge. Throughout I will guide you to a deeper understanding of the remains of Crab Nebula and what information can be calculated.

There are two central questions that followed me throughout.:

First “How can I prove when Crab Nebula was born ?”

Second “How far is it from us, from our earth ?”

The answers to these two, on the surface simple, questions are provided by this paper. Even without prior knowledge we will take a dive into more complicated astronomical knowledge accompanied by mathematical calculations. To achieve this you will find several images and animations, which will accompany you throughout the following events.

Crab Nebula is especially suitable for this purpose, as it was the best fit at the time of research: bright enough and with lots of available sources.

Technical Note:
The animations are embedded right into this PDF and can be played by simply clicking on them.

1.2 Tasks

The task I set out for my self is analyzing the Crab Nebula from birth up until now. On this journey I will explain the following aspects:

- Developing the images in software
- Calculating the birthdate of Crab Nebula
- Calculating expansion rate, size and distance
- Analyzing how accurate the results are

2 Theoretical basics

To understand the whole picture we first have to understand the basics. Explaining what a Supernova is, we first have to understand the evolution of stars. Before that we have to know what its predecessor is and how it stays “alive”. This can only be understood with some basics.

As a refresher:
An atom can be represented in two essential parts, one of them being the nucleus the other one being the surrounding electrons. We are interested in the electrons and the nucleus with its protons and neutrons. Imagine the protons and neutrons as little bubbles in a bigger bubble with the electrons orbiting around the nucleus in circular orbits.

A star like our sun keeps itself alive through two forces cancelling each other out. One of them being gravity and the other fusion. Everything that we know is influenced by gravity[1].

Due to gravity everything gets pulled together:
The more mass in one area, the stronger its gravitational attraction. This indicates, if some objects are in an absolute vacuum, these would get pulled together. In a star these atoms exert an enormous amount of pressure on each other, pulling themselves into a sphere. This results in equilibrium, where an even pressure is built up.

The deeper you go into this sphere the higher the pressure gets. At a certain threshold all the built-up pressure results in the atoms stripping their electrons. With even more pressure the remaining nuclei start to fuse and release energy in the form of heat and light. This force pushes against gravity and holds the star in a stable equilibrium until the fuel runs out[2].

The Hertzsprung–Russell diagram (Fig.1) shows the luminosity increasing as the stars appear higher in the chart and the color of the light increasing as it moves further right. It represents the different star types. Only red giants and supergiants explode in a supernova. Lighter stars, which are in the bottom and middle of the diagram, don’t have enough mass to end in such a spectacle. Crab Nebula once was a red giant, which exploded.

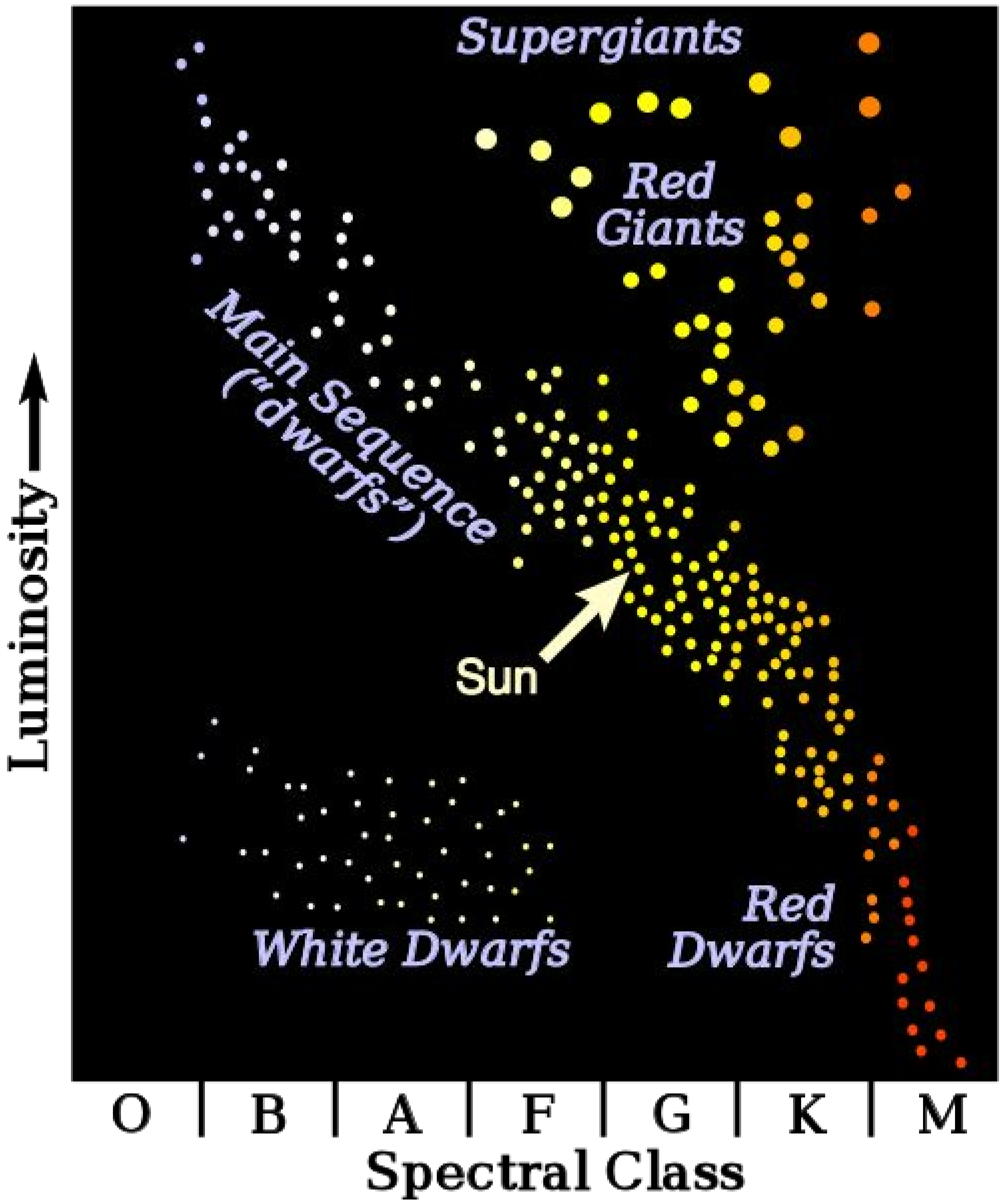

Fig. 1: HR-sparse by User:Rursus 2007-04-30 from https://commons.wikimedia.org/wiki/File:HR-sparse.svg

Running out of hydrogen is the beginning of the end of a star. The hydrogen fuses into helium, after that heavier elements fuse. Subsequently, oxygen fuses into neon, silicon, sulfur and at last into iron[3].

Each heavier element gives of less and less energy than its predecessor, which makes its existing time shorter and shorter. Iron is so heavy, it needs more energy to fuse than it releases at fusion. This means no further fusion will happen from that point on, and the star will die.

The death of a heavy star is the birth of the supernova.

There now is no force opposing the gravitational pull. The surrounding matter pushes everything even closer together until it implodes.

In the center, the now fully iron core, gets squished so far, all the nuclei get pushed onto each other, and the entire space fills up like it were one giant super-nuclei. The entire star implodes, and the outer mass bounces off the core in a giant explosion, and the supernova gets born.

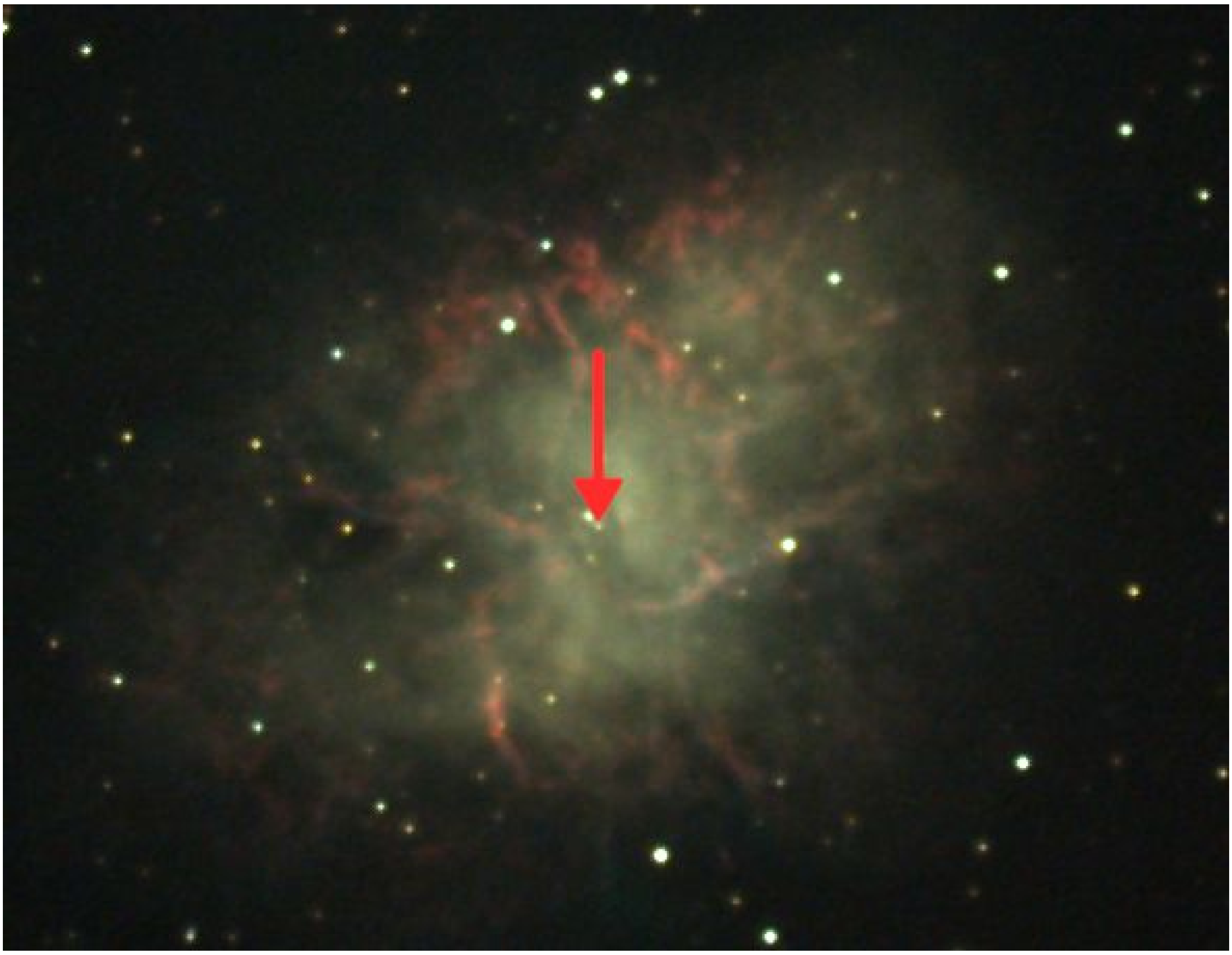

Fig. 2: Crab Nebula by Nathan Mossaad 2021-01-12

The original core now is a new star: A neutron star as it is entirely made out of neutrons, which are parts of a nucleus. In the photo the new star is marked with arrows[4].

This process can be seen in this graphic by following the upper path up to the neutron star:

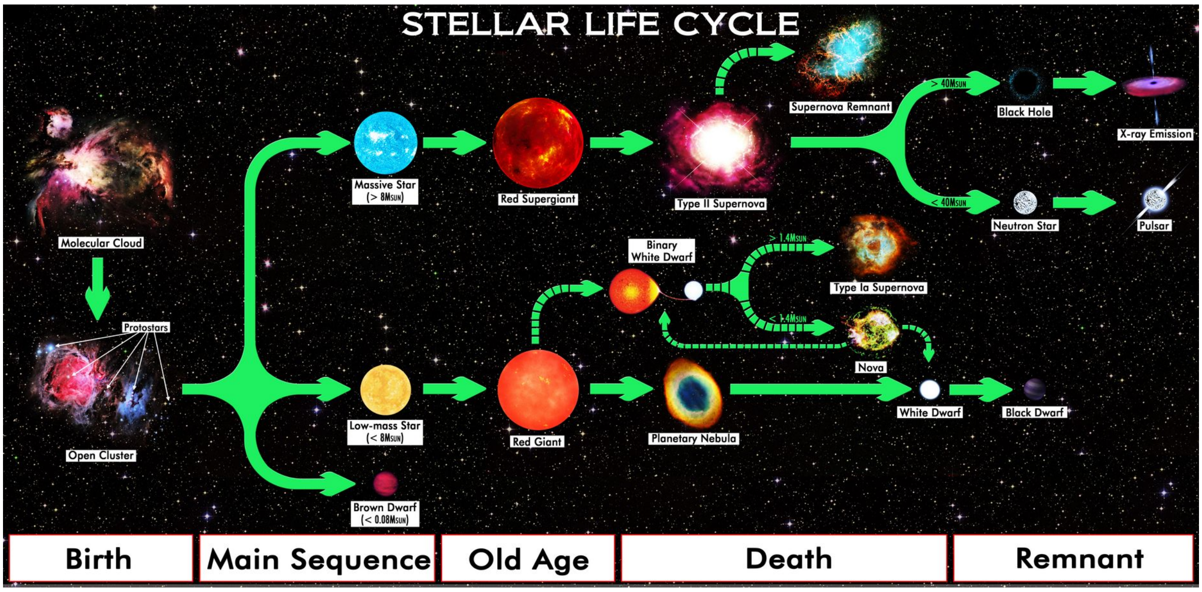

Fig. 3: Star Life Cycle Chart by R.N. Bailey 2017-05-23 from [*https://commons.wikimedia.org/wiki/File:Star_Life_Cycle_Chart.jpg*](https://commons.wikimedia.org/wiki/File:Star_Life_Cycle_Chart.jpg)

3 Photography of the Crab Nebula

3.1 Hardware

The following table contains the hardware that was used for the images:

Camera	STF-8300M with Baader RGB CCD-filter[5]
2nd Camera	ZWO ASI 183 MC Pro ASI183MCPRO[6]
Telescope	Celestron EdgeHD Tubus 0,76m[7]
Telescope type	Schmidt-Cassegrain 0,36m

The setup can be seen in this image:

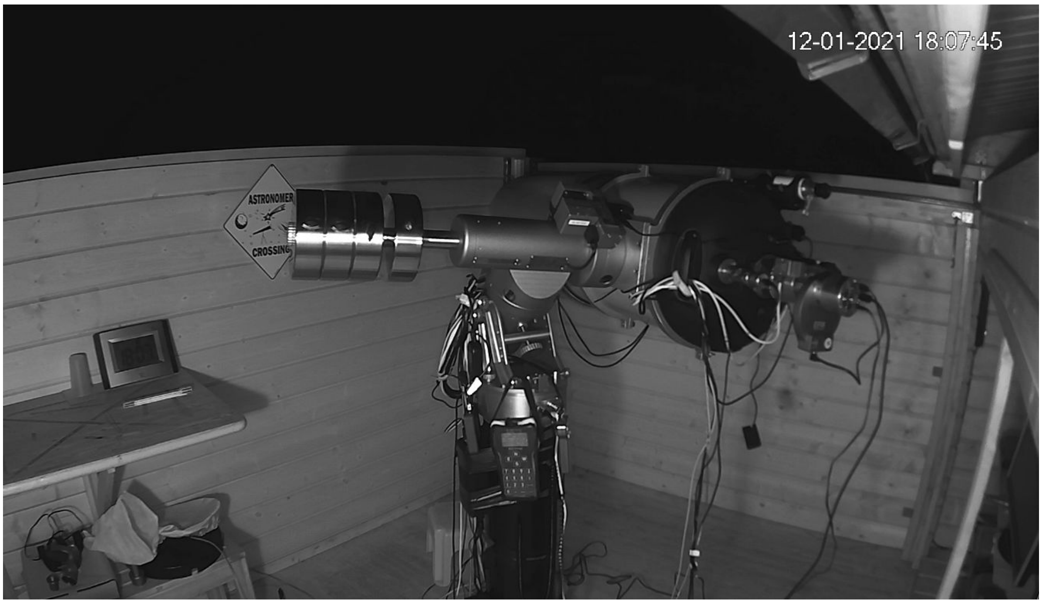

Fig. 4: Used Telescope from Bernd Koch by Bernd Koch 2021-01-12

3.2 Software

3.2.1 10micron Keypad

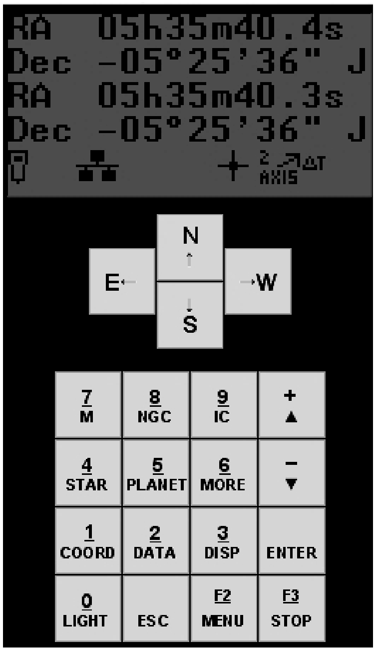

Fig. 5: 10micron Keypad

This software allows us to control the telescope with a PC. It is a mirror of the attached keypad, which allows remote control from inside a house, without giving up monitoring of the telescope. It shows status, and the direction in which the telescope is pointing.

→ This is used when taking images

3.2.2 FocusLynx

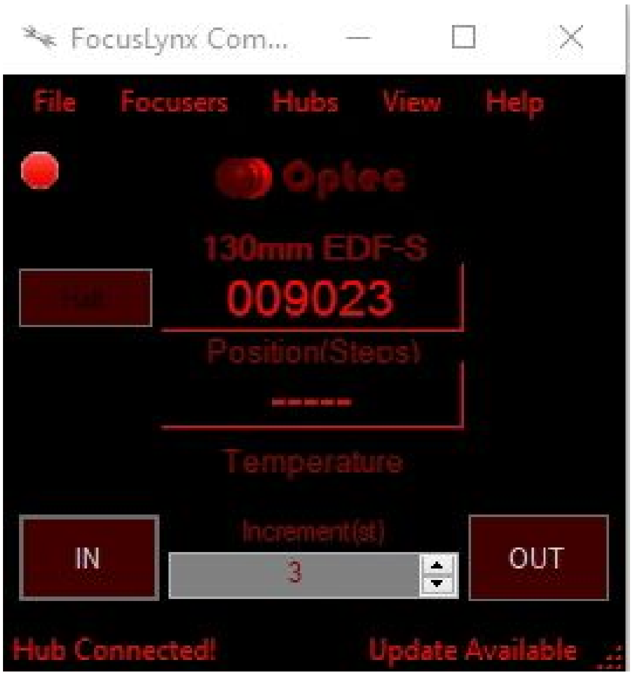

Fig. 6: FocusLynx

This software also connects to parts of the telescope and keeps a given object in focus. Using FocusLynx there is no need of keeping the object focused manually. This is valuable as often as in our case, the object is to dim to see properly without enough illumination time with a camera.

→ This is used when taking images

3.2.3 Dimension 4

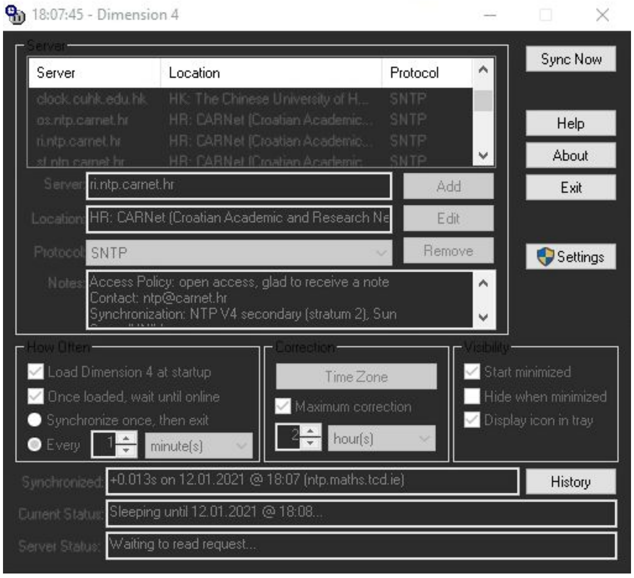

Fig. 7: Dimension 4

This software synchronizes the PC clock with other time-servers to have accurate time. In the image on the left we have an accuracy of ±0.013 seconds. If we see something new, we can compare it with images from other astronomers and with a bit of luck further human knowledge, e.g. for when an object flies by.

→ This is used when taking images

3.2.4 SpecTrack

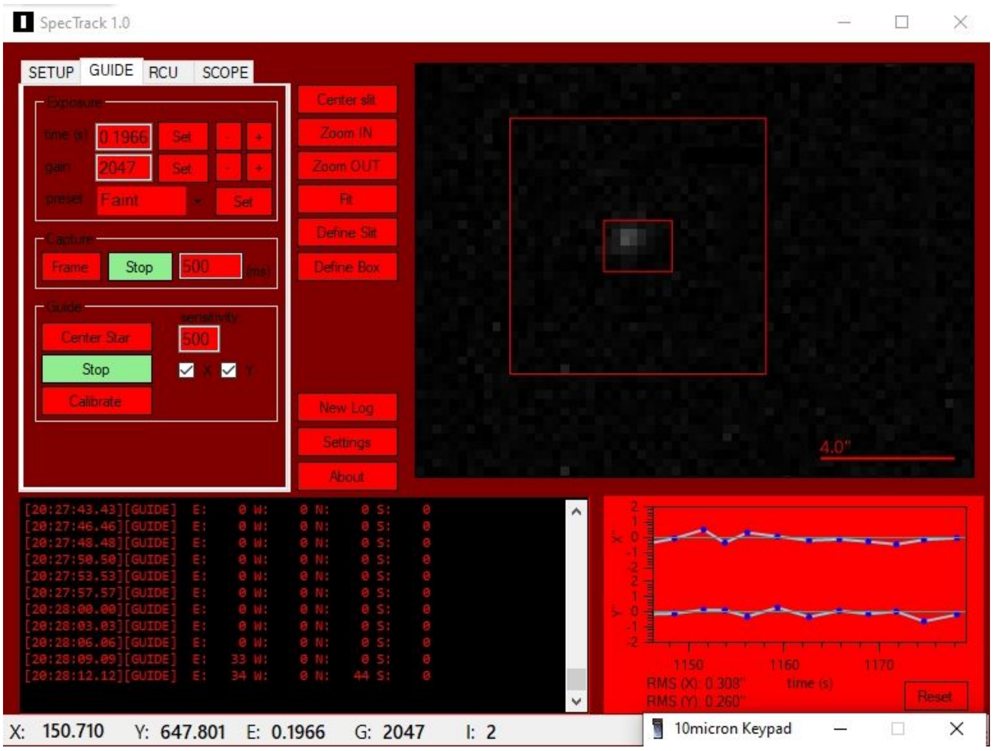

Fig. 8: SpecTrack

This is a simple tool that takes the image of a camera to adjust the telescope to look at an object live while photographing. This results in less jitter than other software can provide by default.

→ This is used when taking images

3.2.5 MaxIm DL

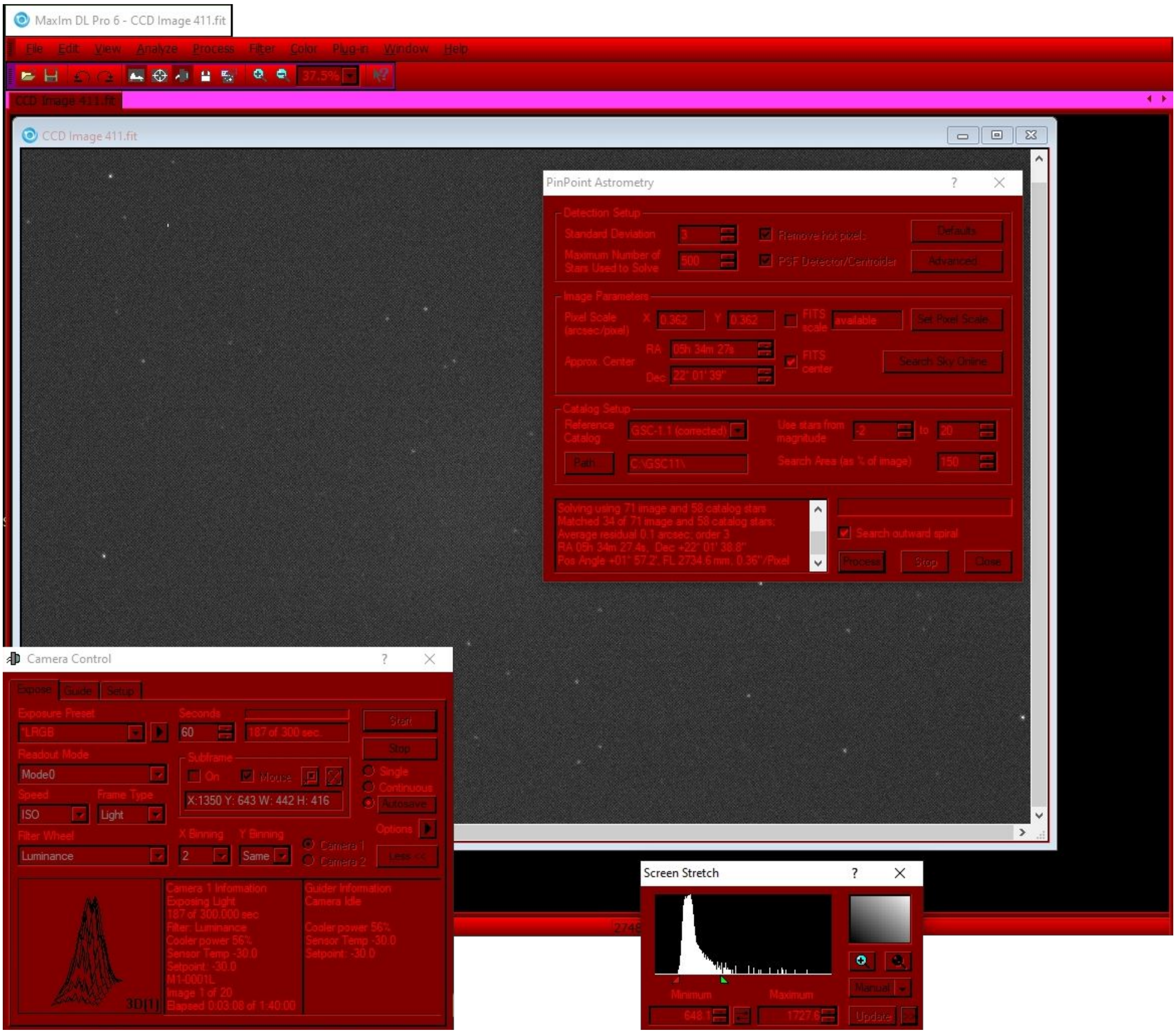

Fig. 9: MaxIm DL

This is the software with which most of the imaging is done. It allows us to tweak most parameters when taking the image. It controls the camera of the telescope with settings for all required parameters. On top of that a multitude of monitoring features for the camera is included.

After taking the images it also allows us to do some processing and further manipulation of the images like aligning images from different dates and animating them.

→ This is used when taking images and afterwards for slight edits

3.2.6 Astroart

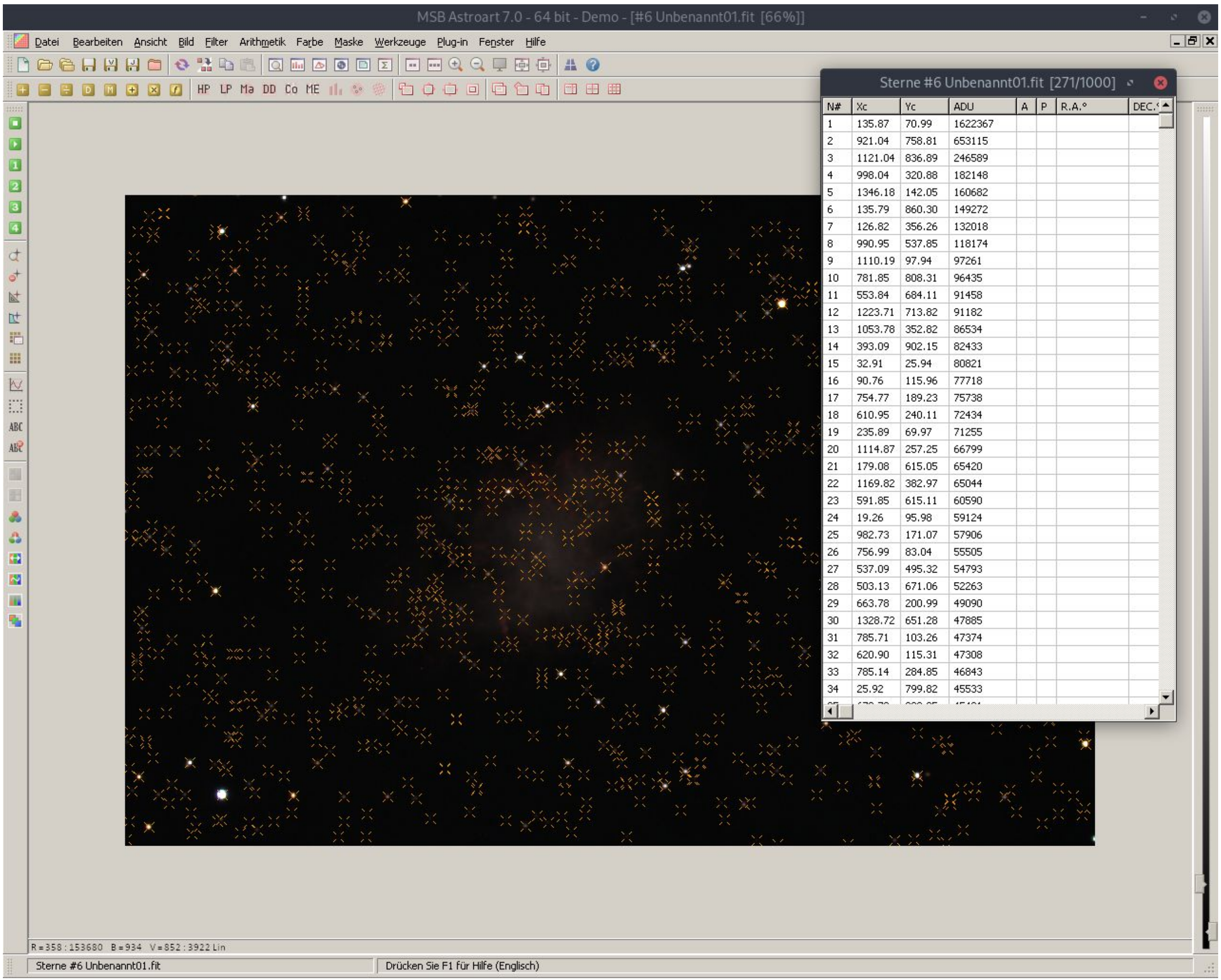

Fig. 10: Astroart

This software is an image editor for astronomical purposes. It has a multitude of features, specific to astronomers. This allows for better editing of the images from MaxIm DL.

→ This is used for photo editing

3.2.7 Stellarium

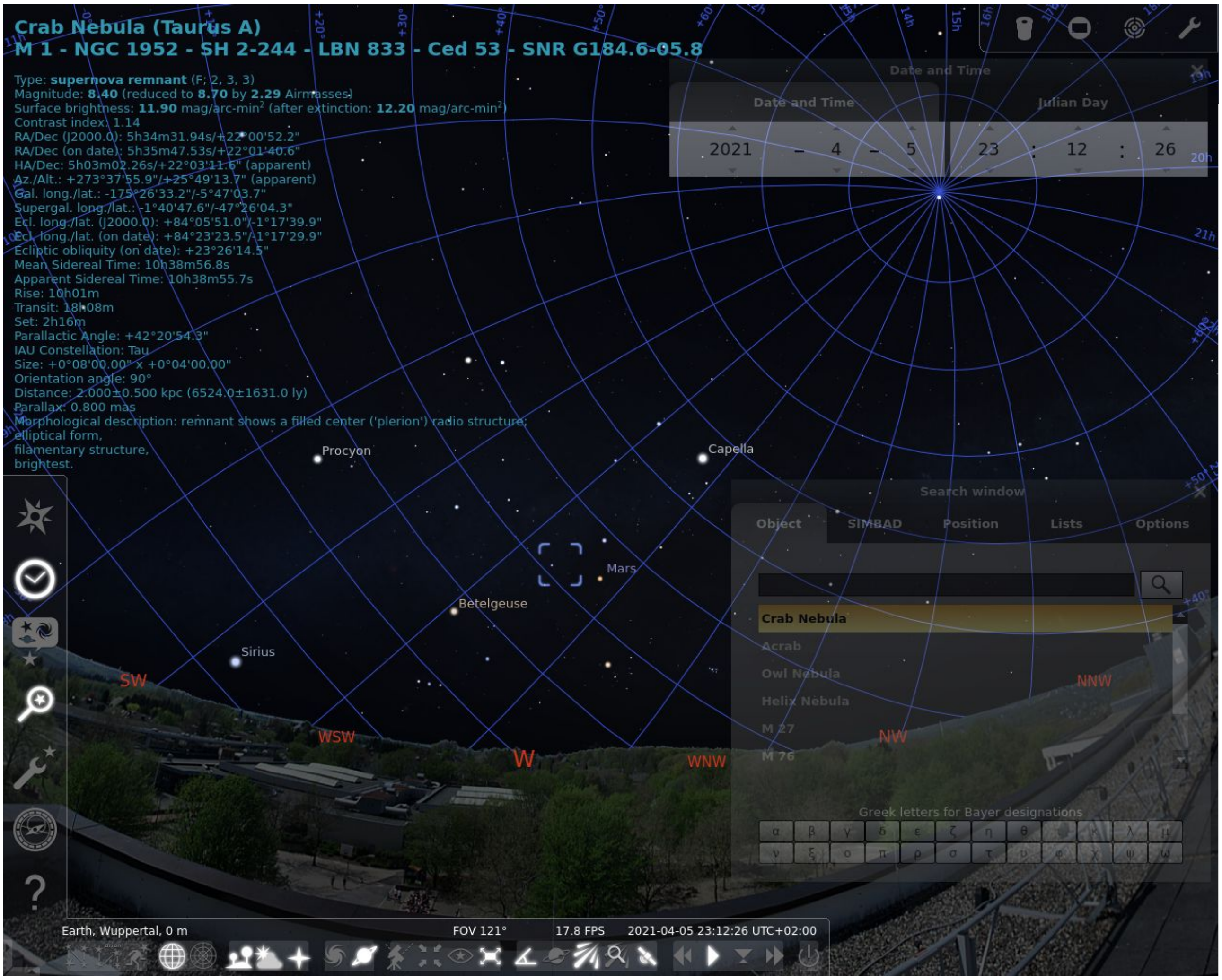

Fig. 11: Stellarium

This software takes on a multitude of functions: It can control where the telescope is pointing, as well as corrections. Furthermore, it has a complete sky map integrated, which allows it to be used for measurements like distance between objects, or the specific coordinates of an object. Most importantly Stellarium shows when and were objects can be seen from earth.

→ This is during the entire process; beginning from first thoughts until deep into research and when taking images

3.2.8 GIMP

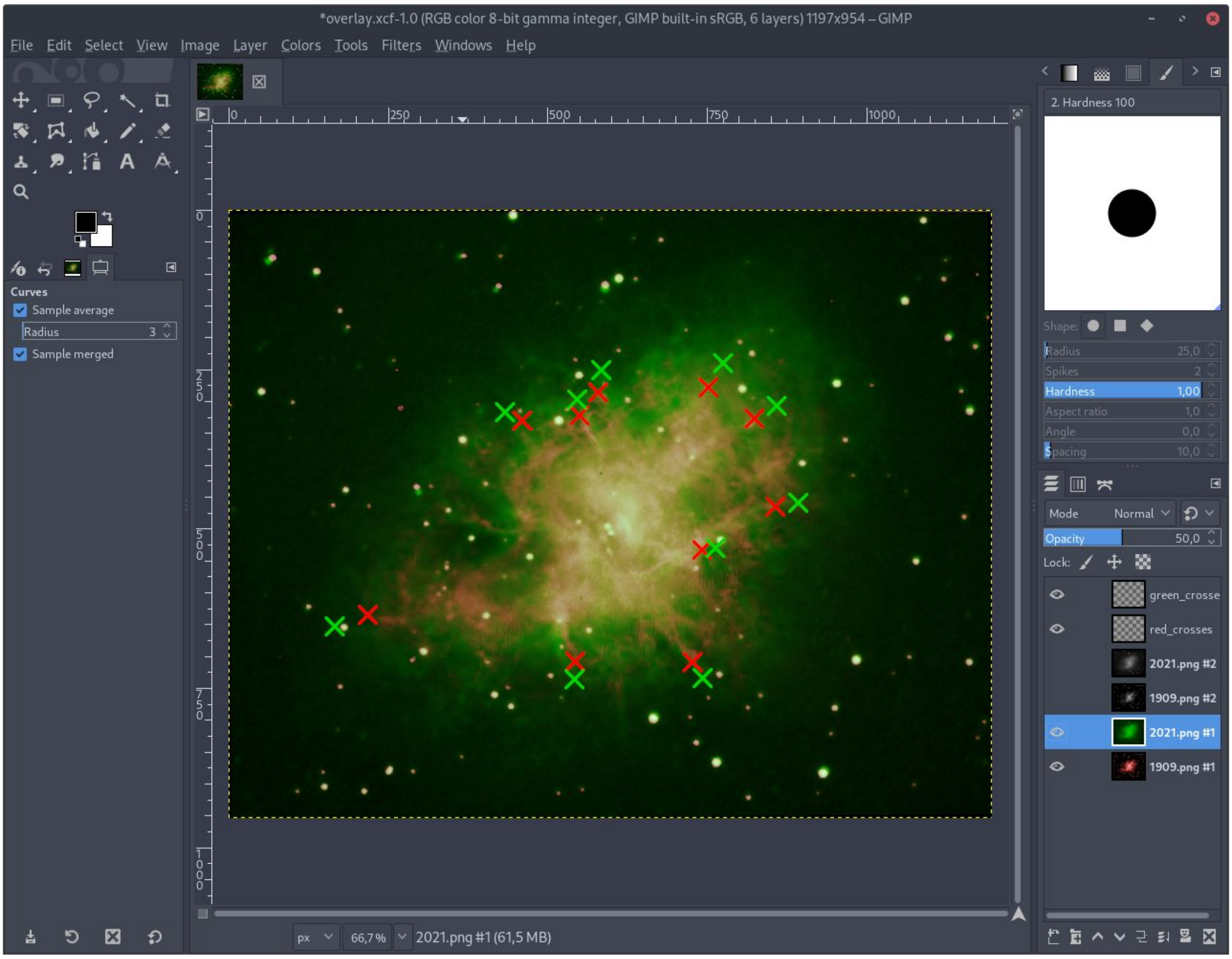

Fig. 12: GIMP

This software is an image editor mainly intended for photographs, but it can achieve a multitude of functions. Here I mainly used it for extracting pixel measurements and visualizations.

→ This is used after imaging for editing

3.2.9 Inkscape

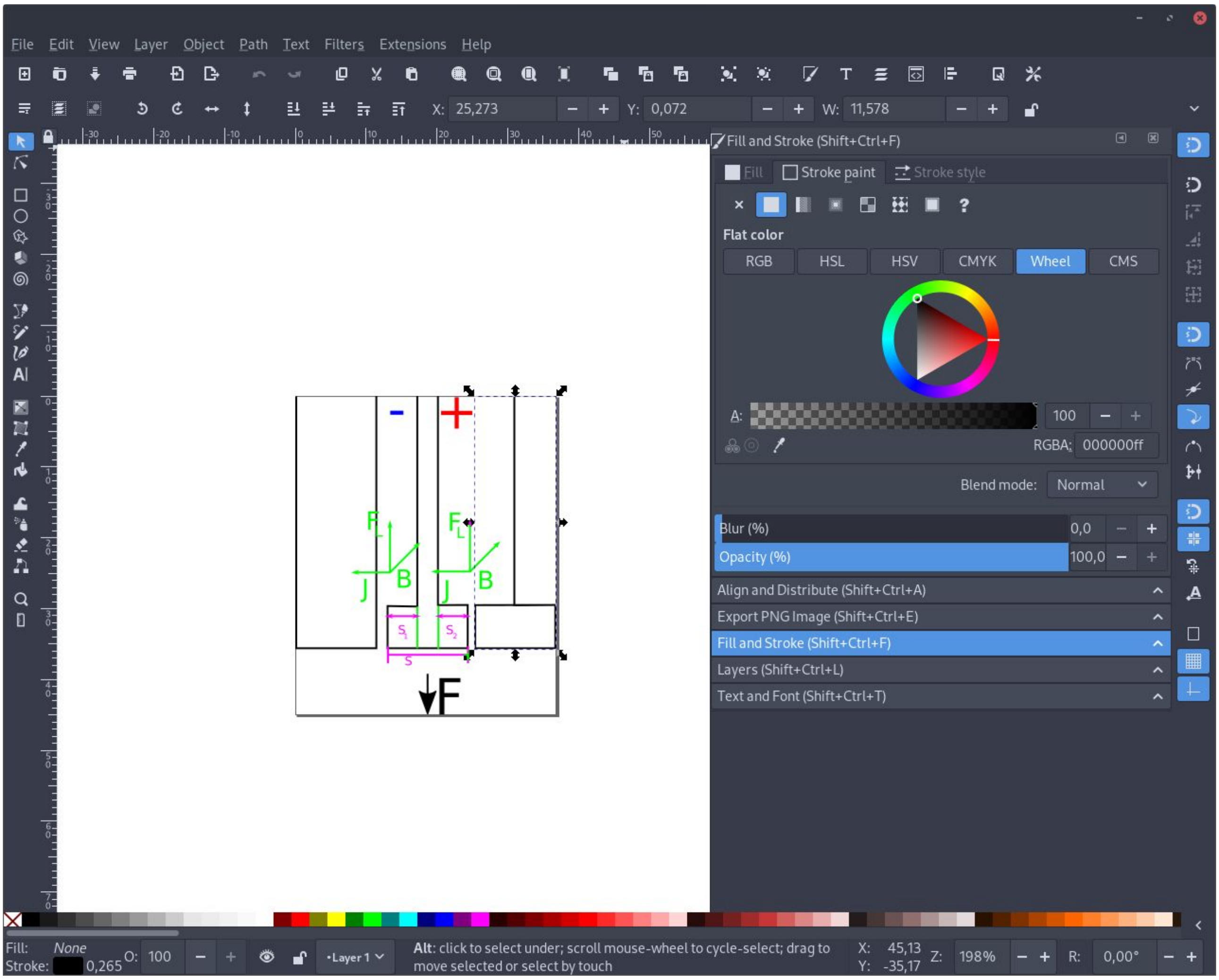

Fig. 13: Inkscape

This software is an SVG (scalable vector graphics) editor, meaning it hasn't got pixels and is unsuited for most applications. It has infinite detail and makes sketches extremely easy.

→ This is used before and after imaging for sketches

3.2.10 LibreOffice Calc

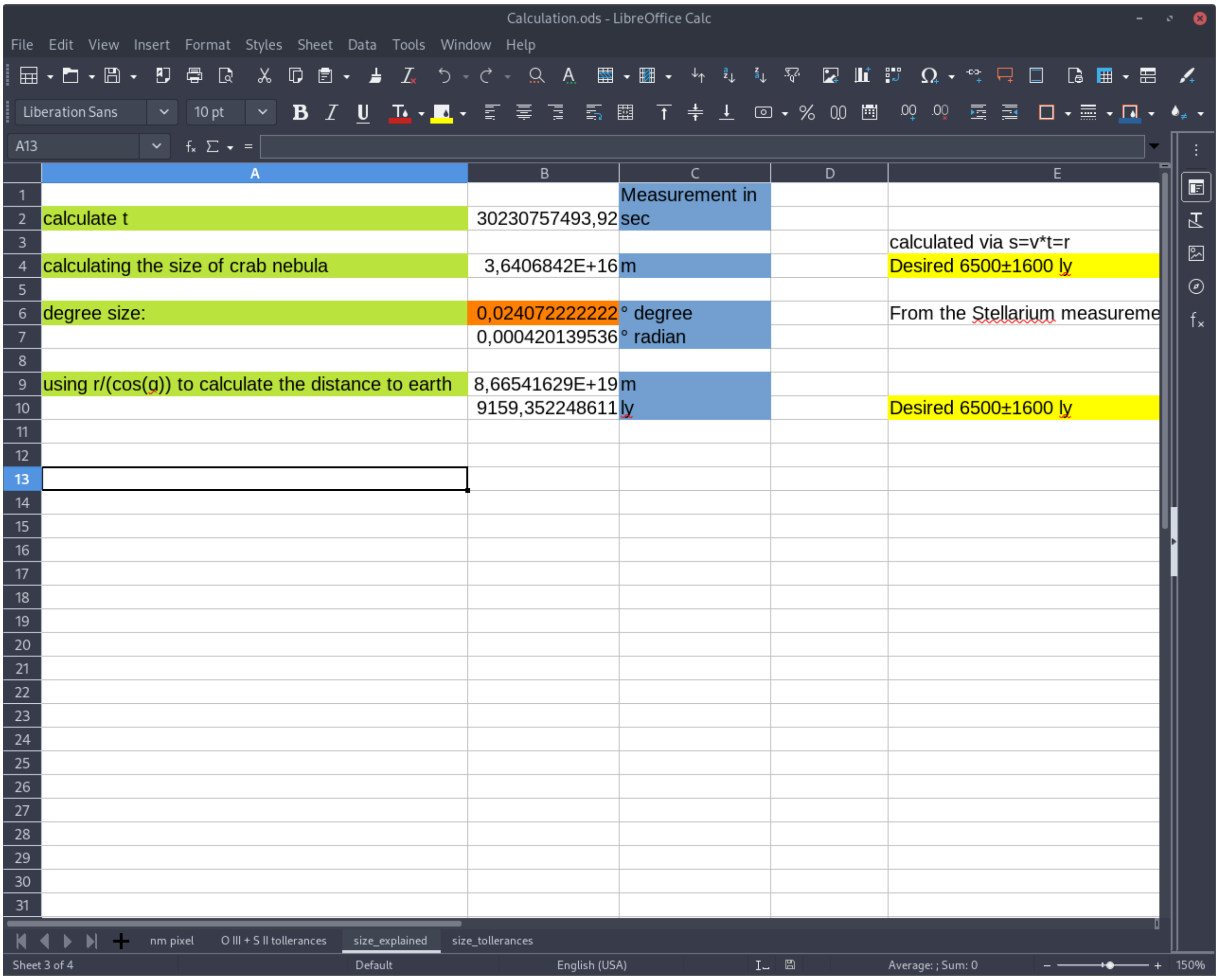

Fig. 14: LibreOffice Calc

This software is an alternative to Excel and with it I executed all of my calculations. In addition, most graphs are generated in LibreOffice Calc.

→ This is used after imaging for calculations and visualizations

4 Color photography

4.1 Taking the raw images

The first step is checking if the equipment is plugged in correctly with all connections being present and most importantly just doing a quick visual check. Then required programs can be started: the first being Dimension 4 with which we will keep track of the time.

With the next program MaxIm DL the temperature of the camera is set and cooling enabled. Cooling is required, because lower temperature reduce image noise.

While the camera is cooling down the telescope controller can be set up. This is done by getting out of the park position and connecting it with Stellarium.

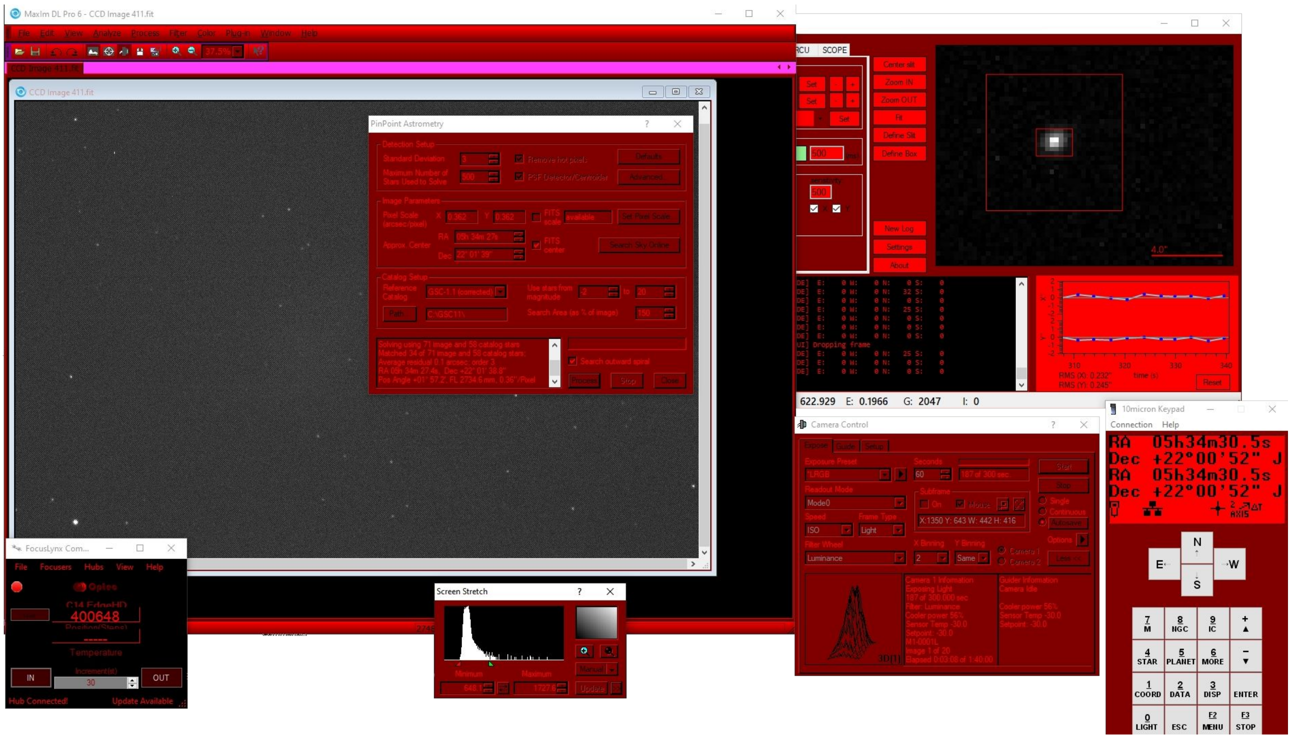

Fig. 15: Screenshot when taking the images on 2021-01-12

After verifying that the telescope and Stellarium are calibrated correctly, a first picture can be taken with the covered lens. This dark image can be used for noise removal. Now the cover gets removed and FocusLynx is started to automatically keep the object the telescope is pointing to in focus.

Finally, the object can be selected in Stellarium, and the telescope gets moved to the correct position. Now SpecTrack is added to control the position of the telescope and correct micro movements to achieve a higher accuracy. After entering exposure time and measurement area for the sensor the photos can be taken in MaxIm DL.

At last the raw images get exported and are ready for further calibration.

All the images from 2021 were made between 19:34 pm and 20:26 pm on 12th January, by remote control of Mr. Koch's Celestron 14 EdgeHD telescope at his private observatory.

4.2 Image processing

After installing Astroart I first imported all .fit files. These are the raw files from the previously captured images.

In order to develop the pictures firstly the color-matrix has to be defined. The raw images only contain the brightness values and not the color of the individual sub-pixels, in this case the Bayer pattern. After that the raw images get converted into a color image and all images get aligned using the star pattern in the background to remove any offset. At last the image gets rotated, as it faces in the wrong direction:

Fig. 16: Crab Nebula colored and rotated

The stars are still big, which can be shrunk to look better by identifying all stars and sharpening these:

Fig. 17: Crab Nebula sharpened stars

Now the picture gets a color correction as the stars should be white, this is done by editing the color-curves:

Fig. 18: Crab Nebula color correction

Finally, some hot pixels remain, which should get removed. This can be done automatically via a predefined function and then the red / green / blue broken pixels get covered up by their surrounding color:

Fig. 19: Crab Nebula final result

4.2.1 Time-lapse

Here are the individual steps in a time-lapse in form of an animation, where all four steps as described previously are shown sequentially (just click on the image):

Fig. 20: Crab Nebula all steps animated https://youtu.be/dG7jq_3Dwt4

5 Data from external sources

5.1 Spectrum

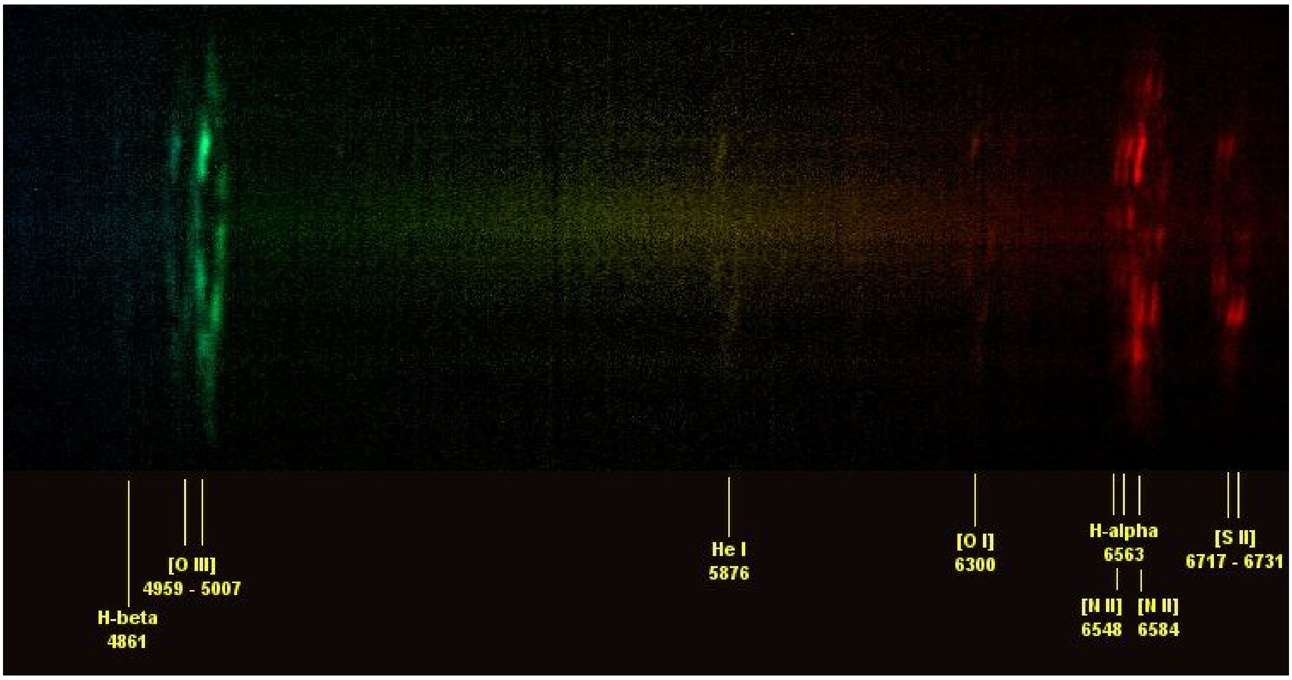

Fig. 21: Spectrum of Messier 1 – The Crab Nebula
by André Rondi, Thierry Maciaszek, Christian Buil 2002-09-02 from
http://astrosurf.com/buil/us/mission2/mission2.htm#SPECTRUM%20OF%20MESSIER%201

This spectrum had to be acquired from an external source, as Crab Nebula has become too dim to create an own spectrum in the conditions of the city with our equipment. I need it for the calculation of expansion velocity, size and distance of the Crab Nebula in chapter 7. The distortion of the lines is because parts of the Nebula expand towards and other parts away from the observer.

5.1.1 General information

Object	M1 Crab Nebula
Date	2nd September 2002
Type	2D spectrum
Exposure time	8 × 3 minutes
Position of slit	Nearly at the center of M1 (crab nebula)
Colors	Artificial, yet represent the covered spectral domain

5.1.2 Reference image of crab nebula

In the image below the reference picture itself can be seen, as it was taken back in 1909 and printed into the paper at NASA. It shows how the Nebula looked back than, which together with the current image can be used to calculate the birthdate of Crab Nebula.

PLATE VII

South

Crab Nebula in *Taurus*

Photographed with 60-Inch Reflector, on Seed 23 plate, October 13, 1909

Exposure 3 hours. Enlargement from negative 8.8 diameters. Scale: 1 mm = 3.″1

Fig. 22: Reference image of Crab Nebula

http://articles.adsabs.harvard.edu//full/1910ApJ....32...26R/0000034P007.html

In the following image the equipment can be seen with which the image was taken. It shows the then new photographic plate-carrier of the 60-inch reflector. This made it possible to keep the photo plate stable enough for long time exposures, which made dim objects possible to photograph.

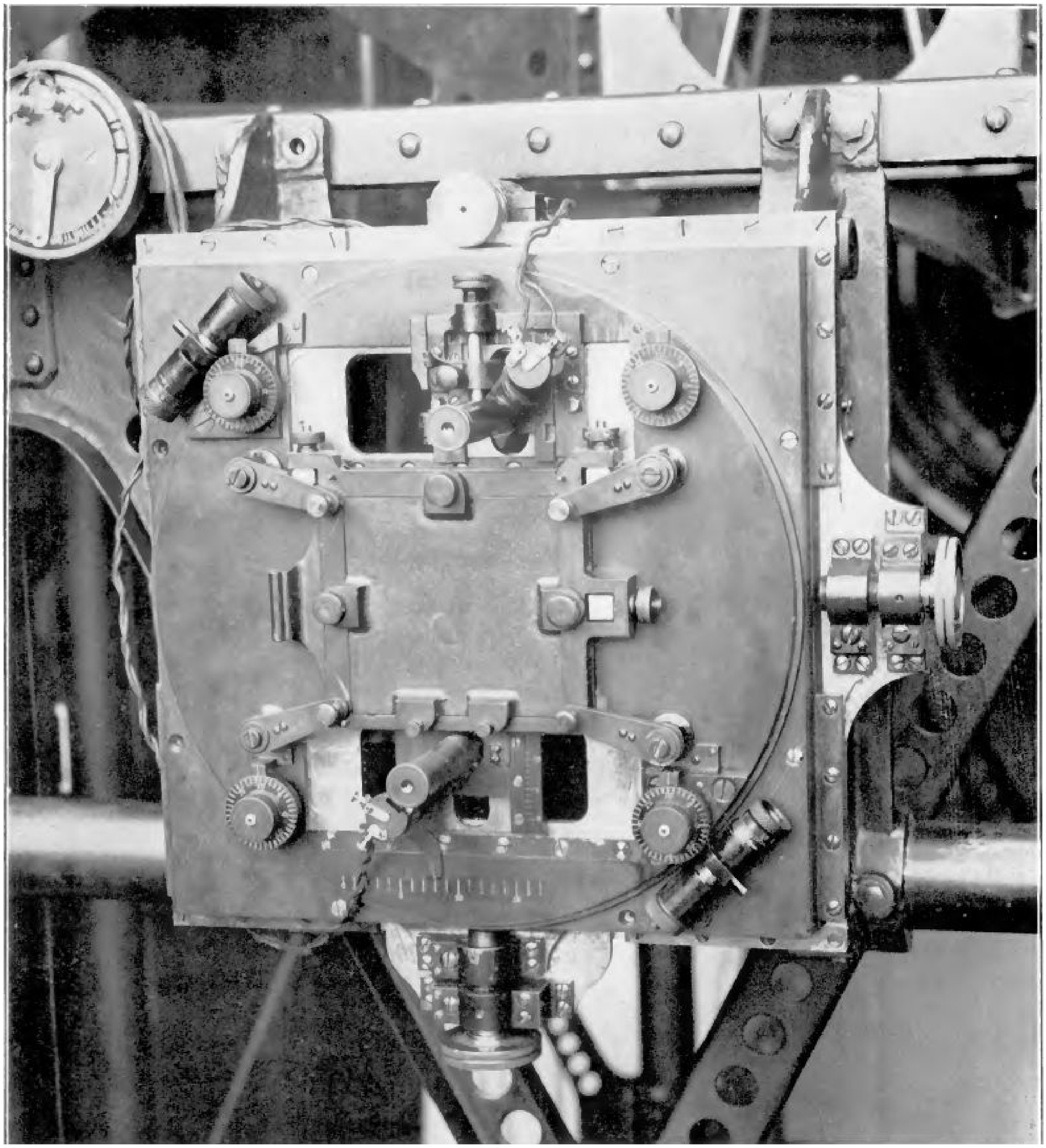

Fig. 23: photographic plate-carrier
http://articles.adsabs.harvard.edu//full/1910ApJ....32...26R/0000030P003.html

Object	M1 Crab Nebula
Date	13th October 1909
Type	Image
Exposure time	3 hours
Technique	Image on photo plate
Reflector Size	60 in ≈ 1,524 m
Seed on plate (Grain)	23
Size	-8.8 diameters
Scale	1 mm = 3.1”

6 Birthdate of Crab Nebula

6.1 Preparation

The first step is taking the previously edited and colored image and turn it back to black and white. This will be the image from the current year.

The image of 1909 is in a PDF, because of which it has to be extracted. I did this with LibreOffice Draw and trimmed it with GIMP straightening it and removing any remaining borders.

Now that the images are ready, they can be prepared to analyze them. After importing them in MaxIm DL, they can be aligned using the alignment function. Marking two stars at the corners of the two Images MaxIm DL aligns, rescales, and stretches them to fit on top of each other. After testing different stars and selecting the best results, they can be exported and further manipulated in GIMP.

Animating the image reveal the images especially well. Movement in the upper left and lower right corners are especially prevalent. Yet this is a good result, as little to none can be found in the middle where crab nebula resides (just click on the image).

Fig. 24: Animation of switching between images https://youtu.be/zWcgKFdqF0A

6.2 Creating more representations

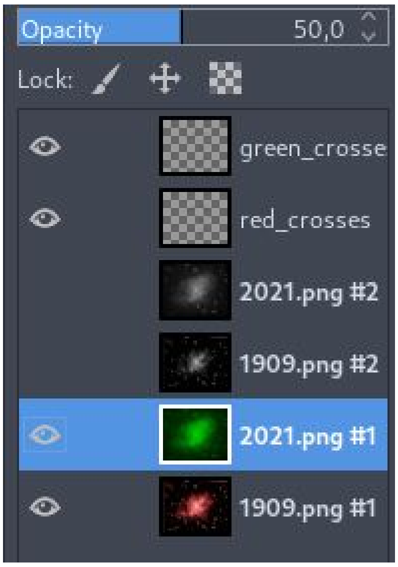

Fig. 25: Layers in GIMP

Using GIMP I created a multi layer representation for further analysis. I began with importing both pictures and coloring them by turning any brightness color into a single color channel: turning the image from 1909 red and the one from 2021 green. Laying these on top of each other and turning the newer picture from 2021 transparent, resulted in seeing them at the same time and keeping the two images animated. On top of these, it was easy to add the markings for the following measurements.

These stacked layers can be seen here (just click on the image):

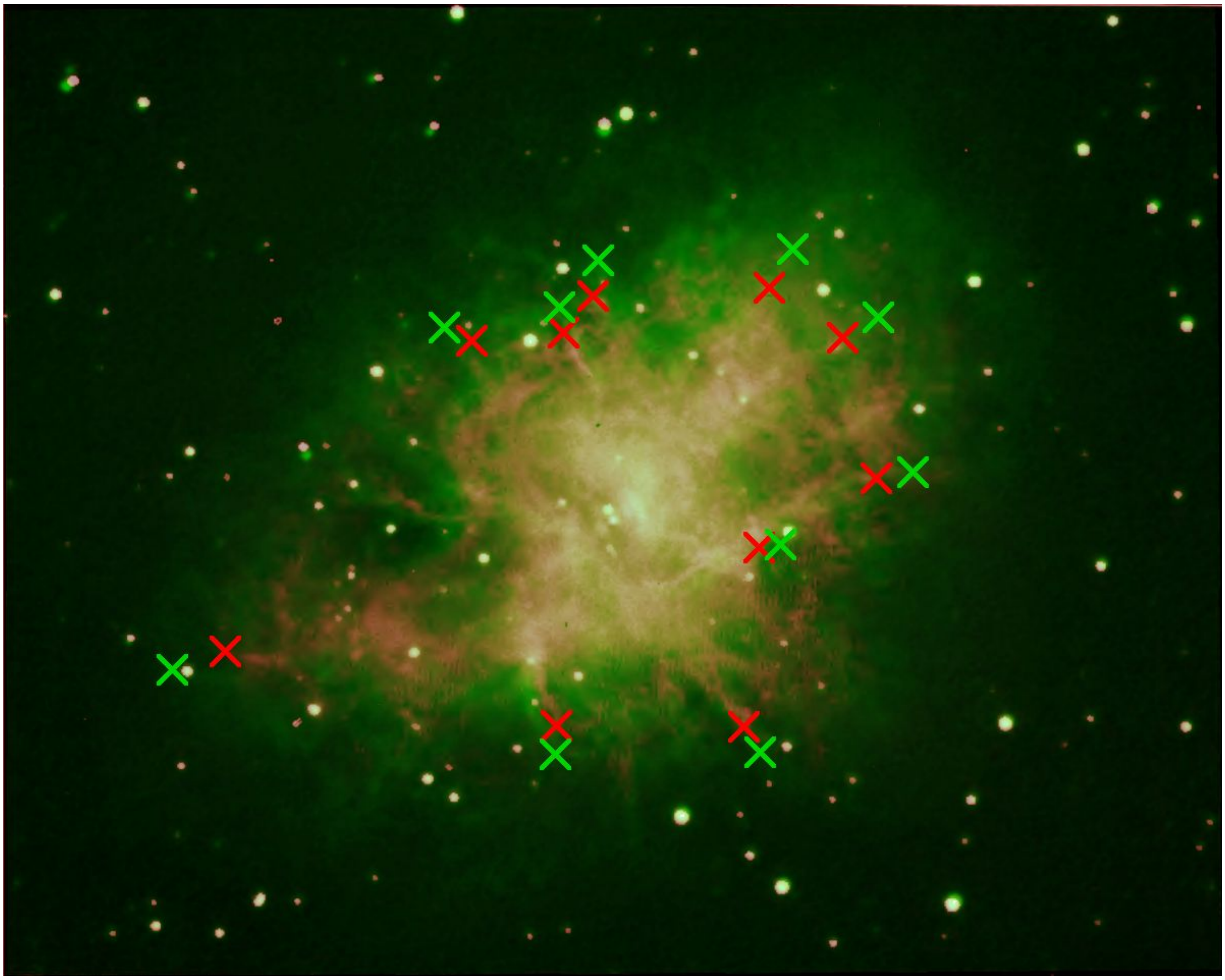

Fig. 26: Layers animated on top of each other https://youtu.be/nHs7Em376FI

6.3 Calculation of the expansion rate of the Crab Nebula

6.3.1 Calculation

In the following with the help of the data from 6.1 and 6.2 I will calculate the birthdate of crab nebula:

6.3.1.1 Reference Points

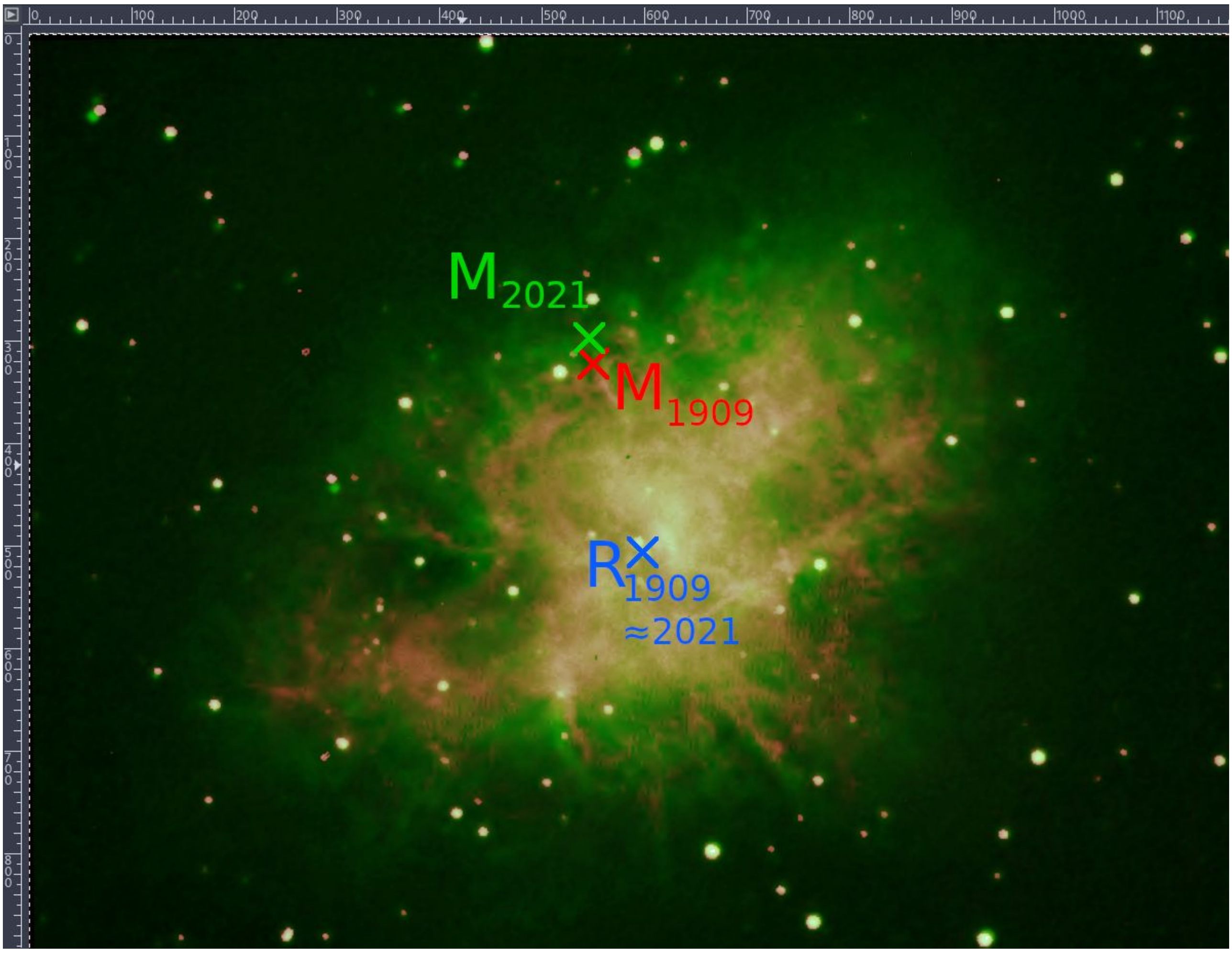

Fig. 27: Marked reference points

The first step is finding a recognizable point in the 1909 image (M_{1909}) and its equivalent in the 2021 image (M_{2021}). The same has to be done with the remnant, meaning remaining neutron star ($R_{1909} \approx R_{2021}$). After recognizing all four points the corresponding coordinates can be noted.

When doing this it is important to know, that the x-coordinates run left to right and the y-coordinates run up to down.

Theory	Practical example (Point 1)
Anno 1909	**Anno 1909**
Measurement point: $M_{1909}=(x_{M1}\|y_{M1})$	Measurement point: $M_{1909}=(550\|322)$
Remnant point: $R_{1909}=(x_{R1}\|y_{R1})$	Remnant point: $R_{1909}=(599\|507)$
Anno 2021	**Anno 2021**
Measurement point: $M_{2021}=(x_{M2}\|y_{M2})$	Measurement point: $M_{2021}=(546\|297)$
Remnant point: $R_{2021}=(x_{R2}\|y_{R2})$	Remnant point: $R_{2021}=(603\|506)$

6.3.1.2 Distance from neutron star to measurement point

To calculate, the distance from remnant point to measurement point first the coordinate systems have to be moved to the same origin. To do that, we set one of the two points (e. g. R_{1909}) to be at the origin and subtract it from the other point. This means, that the distance from origin to the new point is equal to the distance from the measurement point to remnant point. Now the calculation is continued with the following method:

Theory	Practical example (Point 1)
Anno 1909	**Anno 1909**
Origin: $M_{1909}=(0\|0)$	Origin: $M_{1909}=(0\|0)$
New point: $R_{1909}=(x_{R1}-x_{M1}\|y_{R1}-y_{M1})=(x_{P1}\|y_{P1})$	New point: $R_{1909}=(599-550\|507-322)=(49\|185)$
Anno 2021	**Anno 2021**
Origin: $M_{2021}=(0\|0)$	Origin: $M_{2021}=(0\|0)$
New point: $R_{2021}=(x_{R2}-x_{M2}\|y_{R2}-y_{M2})=(x_{P2}\|y_{P2})$	New point: $R_{2021}=(603-546\|506-297)=(57\|209)$

Using Pythagoras theorem the distance itself can be calculated:

Theory	Practical example (Point 1)
Anno 1909	**Anno 1909**
Length: $R_{1909}=\sqrt{(x_{P1})^2+(y_{P1})^2}$	Length: $R_{1909}=\sqrt{(49)^2+(185)^2}\approx 191{,}3792047$
Anno 2021	**Anno 2021**
Length: $R_{2021}=\sqrt{(x_{P2})^2\|(y_{P2})^2}$	Length: $R_{2021}=\sqrt{(57)^2\|(209)^2}\approx 216{,}6333307$

6.3.1.3 Setting up the linear function

Theory	Practical example (Point 1)
The function itself $y=m\cdot x+n$ Here y is the radius of crab nebula in pixels and x the year.	
Anno 1909 $y_1=m_1\cdot x_1+n_1$	
Anno 2021 $y_2=m_2\cdot x_2+n_2$	
Receive the gradient and constant $y_1-m\cdot x_1=y_2-m\cdot x_2$	
Transformed for Gradient: $\rightarrow\ m=\frac{y_2-y_1}{x_2-x_1}$	**Gradient:** $m\approx\frac{216{,}63-191{,}38}{2021-1909}\approx 0{,}2254833$
Transformed for Constant: $\rightarrow\ n=y_1-m\cdot x_1$ n is how big the crab nebula was at 0 AD, according to our function. Of course, it is impossible for it to be negative big.	**Constant:** $n=216{,}63-0{,}23\cdot 2021\approx -239{,}0683544$
Insert into the previous function: $y=m\cdot x+n$	**Insert into the previous function:** $y=0{,}2254833\cdot x-239{,}0683544$

6.3.1.4 Calculating the precise year

Theory	Practical example (Point 1)
Solving for x with y = 0: $0=m\cdot x+n$ $x=\frac{n}{m}$ x is approximately our explosion year, as this is the point in time when the size would	**Solving for x with y = 0:** $0=0{,}2254833\cdot x-239{,}0683544$ $x=\frac{0{,}2254833}{-239{,}0683544}\approx 1060{,}2$

have been roughly 0.

6.3.2 Spreadsheet Analysis

In practice the previously defined, is calculated in LibreOffice Calc and the coordinates are read in GIMP. I have explained it once with the first point, but as they all are the same I don't have to repeat myself.

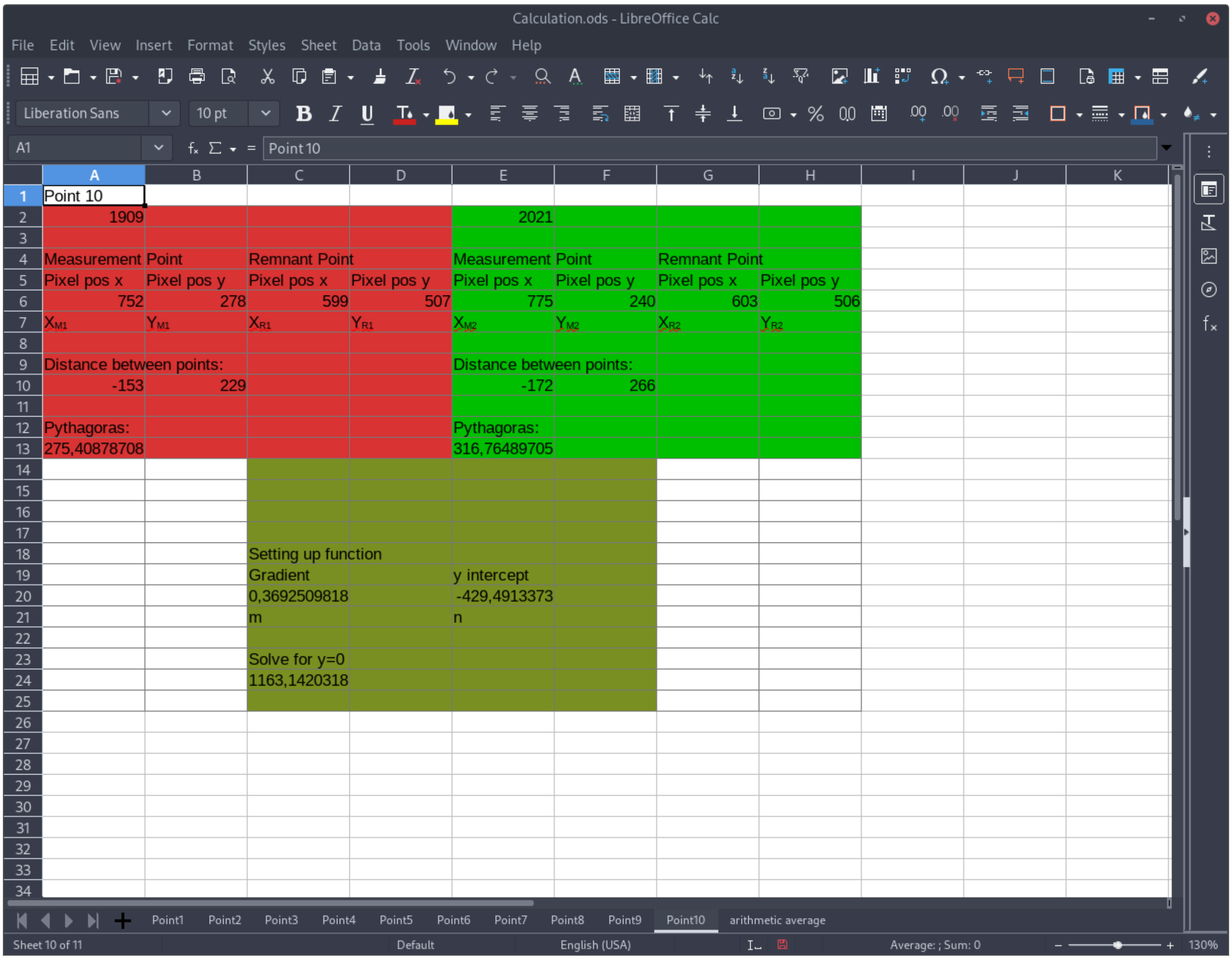

Fig. 28: Animation of flipping through almost identical pages https://youtu.be/ATAxIn75iYE

All the calculations for 1909 are in red and all the calculations for 2021 in green. The calculations for both are in olive green, with the result for each of the separate data points. This means on each sheet the entire previously calculation is performed once, overall ten times (just click on the image).Now that all calculated values are available, the arithmetic average is calculated, which is approximately the year 1060 ± 86,8 compared to the literary value of the 4 July 1054[8] as in the night it was seen by Chinese and Japanese astronomers[9]. More information of the tolerances can be found in the chapter 8 "Analysis of the Results".

7 Expansion Velocity, Size and Distance of the Crab Nebula

7.1 Preparation

The first step is getting a spectrum that I can use for the following calculations. For this I used the aforementioned spectrum. As this image by itself isn't of much use I first had to edit them and interpret them to further analysis.

To better recognize parts of the image I created the yellow and blue markings as seen below. These don't influence the results as only the position of the individual pixels are important and not the brightness. Furthermore, the colors are artificial and are ignored. In the image below the measurement lines from the calculation are already filled in and are thicker than the one pixel that they actually are to increase visibility.

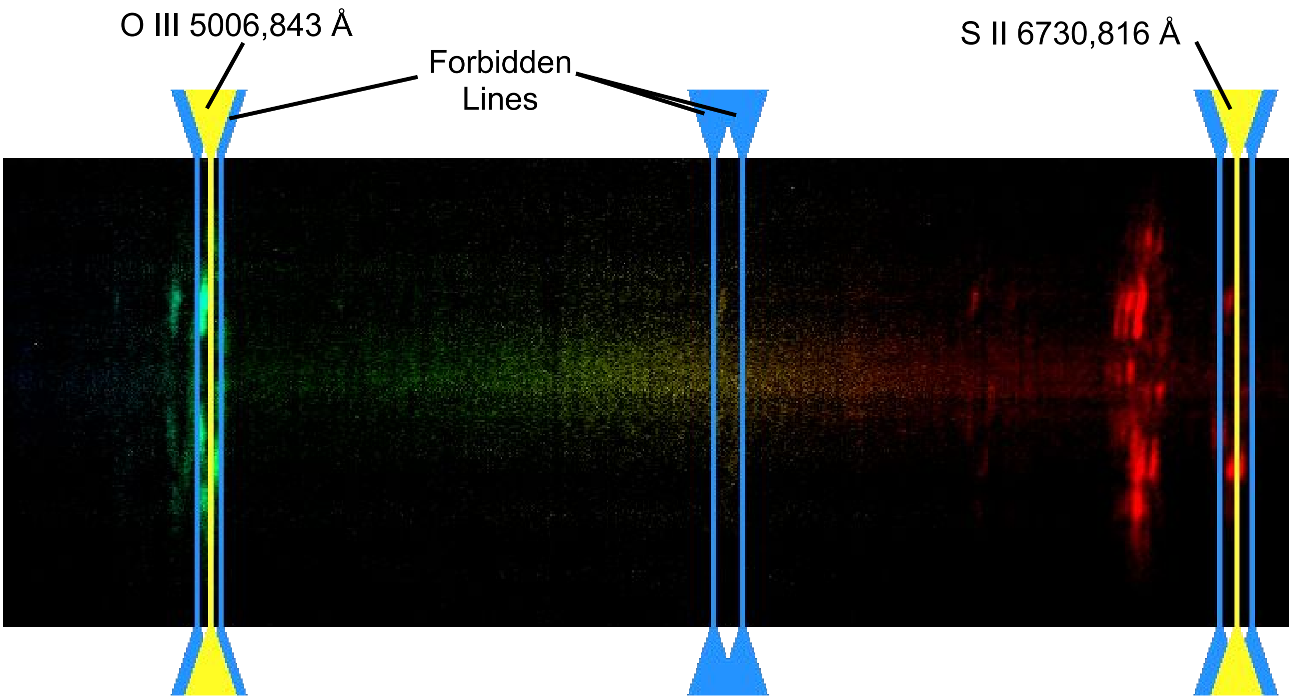

Fig. 29: Spectrum of Crab Nebula with markings
by André Rondi, Thierry Maciaszek, Christian Buil 2002-09-02 from
http://astrosurf.com/buil/us/mission2/mission2.htm#SPECTRUM%20OF%20MESSIER%201

In the image above the left yellow marked line is the O III emission line and the right yellow marked line is the emission line for S II. The each two blue lines mark the forbidden lines for O III, He I and S II are so called as they shouldn't exist and are a result of the Doppler effect.

7.2 Calculations

7.2.1 Conversion Rate

To calculate the conversion rate from pixels to Ångström we have to know the values of each at a precise point. These can be identified by comparing the visible lines with known lines. These are the two yellow lines that are marked in the above image. The one on the left is O III emission line and the right S II line. With this the conversion rate from pixel can be calculated with the following formula:

$$\frac{S\,II\,\AA - O\,III\,\AA}{S\,II\,pixel\,position - O\,III\,pixel\,position} = \frac{6730{,}816 - 5006{,}843}{721 - 121} \cdot \frac{\AA}{pixel} = 2{,}873288\bar{3} \cdot \frac{\AA}{pixel}$$

To calculate the corresponding Å valuables for each position we need the Å value at the beginning position of the spectrum. This can be calculated with the following expression:

$$2{,}873288\bar{3} \cdot \frac{\AA}{pixel} - (5006{,}843\,\AA \cdot 121\,pixel) \approx 4659{,}1751\,\AA$$

Using this it is possible to set up a linear function, which can be used to calculate the corresponding values:

$$2{,}873288\bar{3} \cdot \frac{\AA}{pixel} \cdot pixel \; + 4659{,}1751\,\AA = Corresponding\,Value\,\AA$$

7.2.2 Expansion Rate

To calculate the size of the nebula and the distance to earth the expansion speed is needed. This can be calculated with the help of the Doppler effect. It states that if the emitter of a wave moves towards the observer, the wave will decrease in wavelength. The opposite happens when the source is traveling away from the observer, as the wave length will increase.

This effect can be seen in the spectrum: The left blue line is the line from the part of crab nebula moving towards earth. The right line is the opposite coming from the part moving away from earth, with the middle part being emitted from those parts that are not accelerating towards or away from earth. This understanding requires that the nebula is expanding in all directions evenly and with the same speed.

The three parts are shown visually in the diagram down below, these correspond with the emission lines and forbidden lines on the right:

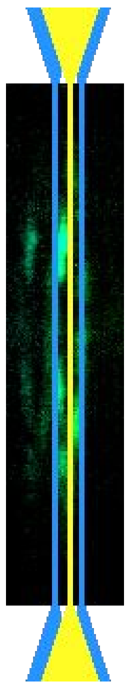

Fig. 30: The O III *emission line*

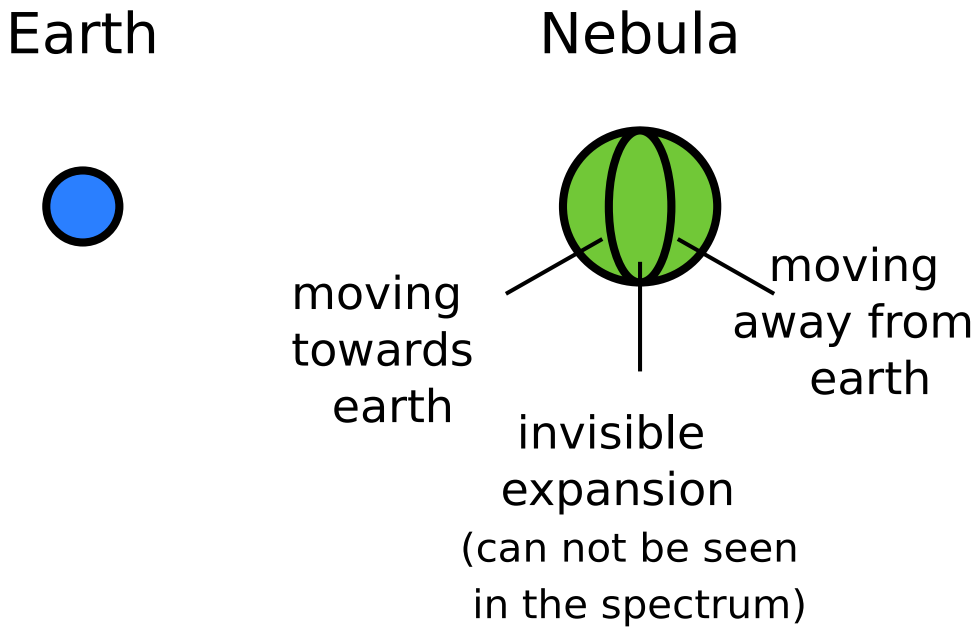

Fig. 31: Diagram of where the spectral lines originate (not to scale)

7.2.2.1 Getting the data

Theory	Practical example for O III
Required Points:	**Required Points:**
$\lambda 0$ → Middle of Borders (yellow line)	$\lambda 0 = 121\ px$
$\lambda 1$ → Left Border (blue line)	$\lambda 1 = 113\ px$
$\lambda 2$ → Right Border (blue line)	$\lambda 2 = 127\ px$
	Converted to Å:
	Via the previously calculated function
	$\lambda 0 \approx 5006{,}84\ Å$
	$\lambda 1 \approx 4983{,}85\ Å$
	$\lambda 2 \approx 5024{,}08\ Å$

7.2.2.2 Calculating the Expansion Velocity

Theory	Practical example for O III
Calculating the difference:	**Calculating the difference:**
$\Delta\lambda = \lambda 2 - \lambda 1$	$\Delta\lambda \approx 5024{,}08\ Å - 4983{,}85\ Å \approx 40{,}23\ Å$

The speed itself: $c = lightspeed$ $v = \frac{(\Delta\lambda \cdot c)}{(\lambda 0 * 2)}$	**The speed itself:** $c = 299792458\,\frac{m}{s}$ $v = \frac{(40{,}23\,Å \cdot 299792458\,m)}{(5006{,}84\,Å * 2\,s)} \approx 1204298{,}04\,\frac{m}{s}$ $\approx 1204{,}3\,\frac{km}{s}$

7.2.3 Size and Distance

With the help of the now gathered and calculated data it is possible to calculate both size and distance.

7.2.3.1 Time since Crab Nebulas birth

Theory	In Practice
In years: $t_{years} \approx time\,of\ picture - time\,of\,birth$ **In seconds:** $t_{seconds} \approx t_{years} \cdot 60 \cdot 60 \cdot 24 \cdot 364{,}24425$	**In seconds:** $t \approx (2021{,}2 - 1060{,}6) \cdot 60 \cdot 60 \cdot 24 \cdot 364{,}24425$ $\approx 30230757493{,}92\,s$

7.2.3.2 Size of Crab Nebula

Theory	In Practice
$r = v \cdot t = radius$ v is the speed calculated previously t is the time since Crab Nebulas birth	$r \approx 1204298{,}04\,\frac{m}{s} \cdot 30230757493{,}92\,s$ $\approx 3{,}640684 \cdot 10^{16}\,m$

7.2.3.3 Distance to earth

Measuring the distance between the edge of the nebula and the remnant in Stellarium.

Theory	In Practice (Point 1 and O III)
Converting degree to radian:	**Converting degree to radian:**
$rad=\frac{deg\cdot\pi}{180}$	$\frac{0{,}024072°\cdot\pi}{180}\approx 0{,}000420139\,rad$
Calculate the distance:	**Calculate the distance:**
$\frac{\text{Radius}}{\tan(rad)}$	$\frac{3{,}640684\cdot 10^{16}\,m}{\tan(0{,}000420139)}\approx 8{,}66541\cdot 10^{19}\,m$ $\approx 8{,}66541\cdot 10^{19}\,m$

Now that all calculated values are available, the arithmetic average of the distance to earth is approximately 7129 ±1709 lightyears[10]. More information of the tolerances can be found in the chapter 8 “Analysis of the Results”.

8 Analysis of the Results

8.1 Birthdate of Crab Nebula

First of all the standard deviation is about 87 years, with a minimum of 919 years and a theoretical maximum of 1173 years. All values are below in the table and integrated in a graph.

Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
1060,25	1173,78	1111,31	1015,24	1147,77	966,3	1044,32	1005,06	919,15	1163,14

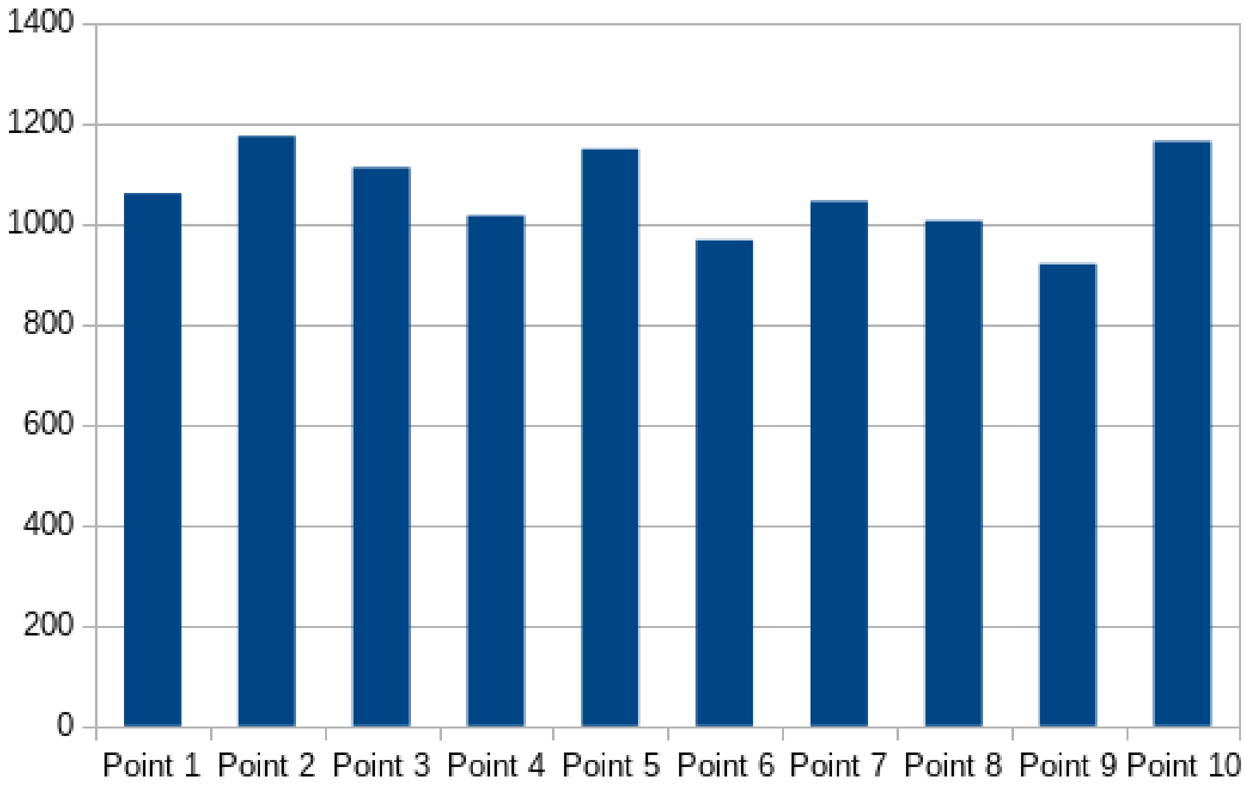

Fig. 32: Distribution of the resulting birthdates of Crab Nebula

These show the difference between results, when calculating a percentage. It means that my result should deviate up to about 8,1%. On top of that there could be a higher deviation, as I could have even more error than anticipated.

8.2 Expansion Velocity, Size and Distance of the Crab Nebula

8.2.1 Explanation

The first opportunity for error is when setting up the function where I could be 1 pixel off in each direction.

After that the same can happen for the O III line and on top of that I used a second line, the S II line, to exclude even more error. After that I got these values:

	+1	-1
O III	16,73% more	12,55% less
S II	11,81% more	09,56% less

Finally I used all ten points from the initial calculation, which were all used to calculate the final results:

		tolerances 1		
1.png				Measurement in
	calculate t		30230757493,92	sec
	calculating the size of crab nebula		3,64068419521258E+016	m
	degree size:		0,024072222222222	° degree
			0,00042013953605	° radian
0,024072222222222	using r/(cos(α)) to calculate the distance to earth		8,66541629295497E+019	m
			9159,35224861059	ly

Fig. 33: Single calculation for the first point and no tolerances

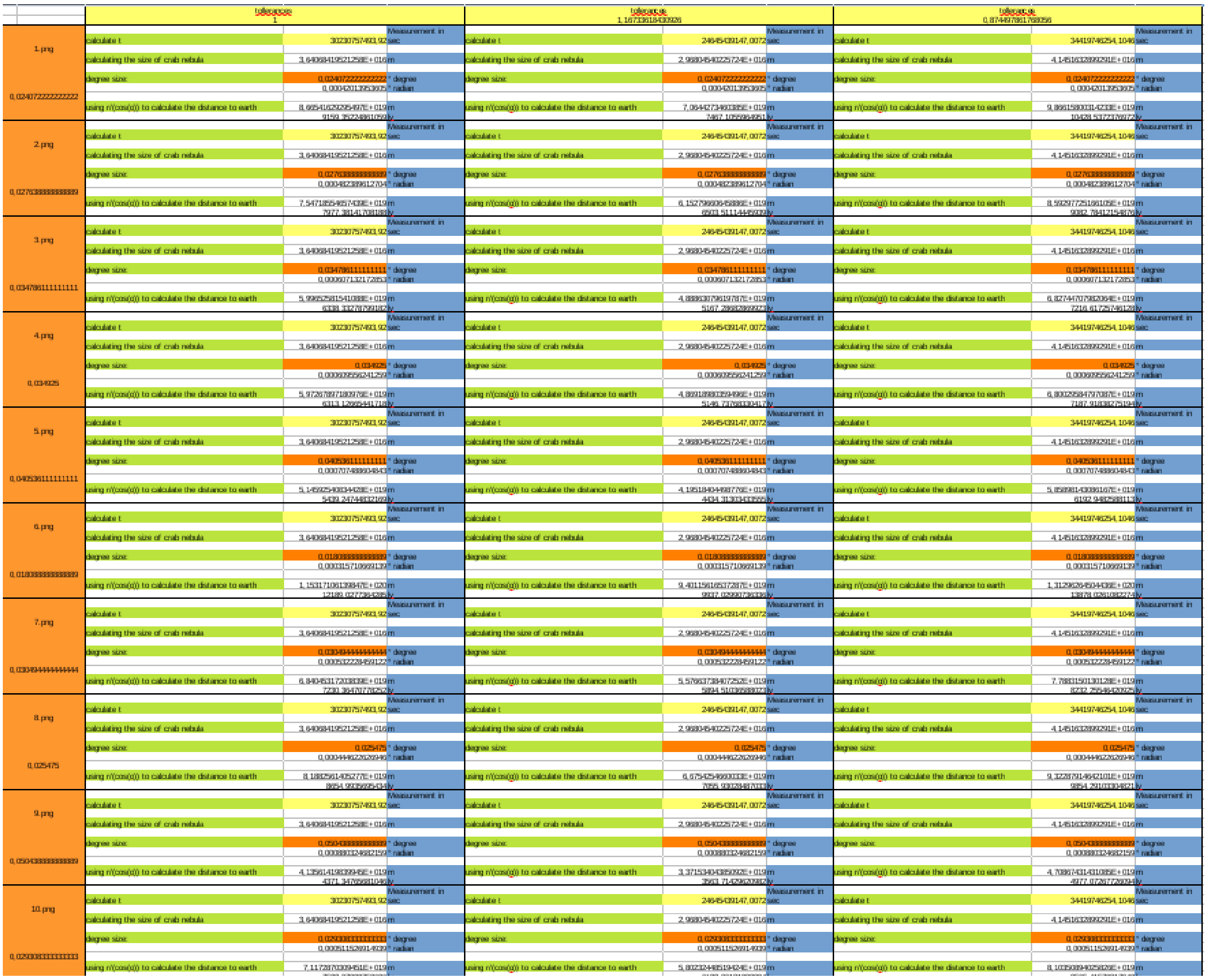

Fig. 34: Calculations for all points

After filtering out all divergent values I ended up with 27 values with a much higher accuracy:

Before	After
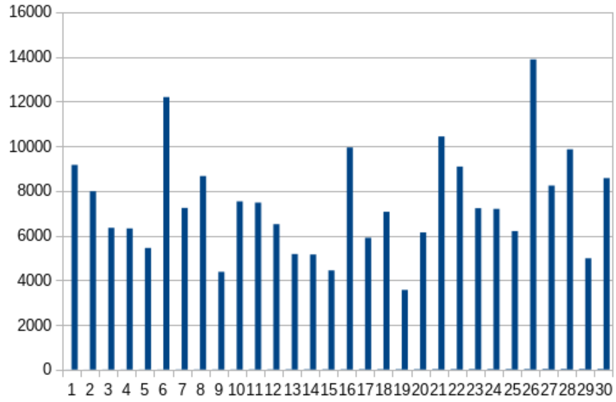	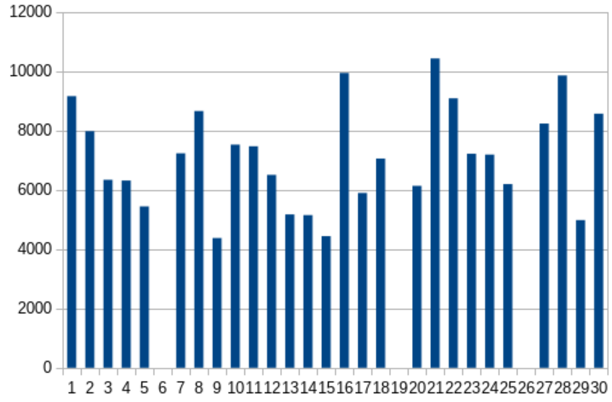
Fig. 35: Distribution of the non filtered Results	*Fig. 36: Distribution of the filtered Results*

This then finally resulted in a distance to earth of approximately 7129 ±1709 lightyears.

When calculating a percentage it means that my result should deviate to about ± 24%. This is quite a bit higher than calculating with the birthdate, which is to be expected, as there a lot more variables to consider and more sources that can increase inaccuracy.

Compared to the value of $6500 \pm 1600\, ly$ [10], my results overlap yet aren't very close to each other. This is due to all of the inaccuracy described above and maybe more as a different method to calculate the distance might be used in the sources.

8.2.2 The data

Here you will find all measured values listed along with standard deviation and official results. For a better comparison the average and official results are marked in yellow.

	Velocity in $\frac{km}{s}$
For O III	1204
+1	1032
-1	1377
For S II	1216
+1	1087
-1	1344
Average	1210
Standard Deviation	± 136,18

→ literary value 1150 $\frac{km}{s}$ [11]

	Nebula radius in ly
Standard value	3,8482
+1	3,1372
-1	4,3814
Average	3,7889
Standard Deviation	± 0,6242

→ literary value 5,5 ly [12]

I used only three values for as it is only a byproduct of the distance, which I wanted to calculate. These are the calculated value and the other two beeing worst case values in both directions.

Distance in ly

Distance between remnant and measurement point	tolerances		
	1	1,16733618430926	0,874497861768056
0,0241	9159,35	7467,11	10428,54
0,0276	7977,38	6503,51	9082,78
0,0348	6338,33	5167,29	7216,62
0,0349	6313,13	5146,74	7187,92
0,0405	5439,25	4434,31	6192,95
0,0181	12189,03	9937,03	13878,03
0,0305	7230,36	5894,51	8232,26
0,0255	8654,99	7055,93	9854,29
0,0504	4371,35	3563,71	4977,07
0,0293	7522,98	6133,06	8565,42
Average		7129	
Standard Deviation		± 1709	

→ literary Value 6500 ± 1600 ly [10]

9 Conclusion

To summarize: I reached my goals that I defined at the beginning. It was possible for me to acquire previously unknown knowledge I was interested in, meaning the birth of Crab Nebula. I believe that I could transfer my new knowledge that I learned in this project work to you, the reader, without overwhelming you.

I had to acquire lots of data and had to process all of it with the help of LibreOffice Calc, which was a lot of work but at the end of the day I'd say it was clearly worth it. The more values I used the more accurate the results got, this was especially apparent when calculating the birthdate of Crab Nebula. Nevertheless, the accuracy is pretty good with my available tools and data. Compared to the literary target values mine weren't that much off.

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12 Final declaration

I ensure that I have written the paper independently, that I have not used any sources or aids other than those indicated, and that any passages in the paper that I have taken from other works, either verbatim or in spirit, have been marked as borrowed, indicating the source in each case.

Ich versichere, dass ich die Facharbeit selbstständig verfasst, dass ich keine anderen Quellen und Hilfsmittel als die angegebenen benutzt und die Stellen der Facharbeit, die ich anderen Werken im Wortlaut oder dem Sinn nach entnommen habe, in jedem Fall unter Angabe der Quelle als Entlehnung kenntlich gemacht habe.

Solingen, 17 May 2021

Nathan Mossaad